A SIMPLIFICATION IN ELEMENTARY TRIGONOMETRY.

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It is generally admitted that consideration should be paid rather to the normal student who forms the bulk of the class than to those few who find the subject particularly easy or particularly hard.

The normal student who studies trigonometry learns numerous formulæ and forgets them very soon after his course has been completed. If on some later occasion he finds an opportunity for applying the knowledge which he once possessed, his recollections have probably merged into confused memories of endless formulæ and he finds it impossible to extricate that one for which he has particular need. To a certain extent this is inevitable; but there is one change in the mode of presentation usually adopted in the text-books which would do something to lessen this difficulty. If a single method can be made to replace a number of processes there is, firstly, less to remember and, secondly, more practice in using what is retained and a double gain is made.

The method which it is the object of this article to advocate has reference in particular to the relations between the trigonometrical ratios of a single angle and their use in the proofs of identities, the solution of equations, and the reduction of complicated trigonometrical expressions to their simplest form. Of these three classes of problems the last is at once of the greatest practical importance and that one in which the method is of the greatest use.

The method is to translate each trigonometrical problem into an algebraic one.

(i) Select that one of the six ratios which occurs oftenest and denote it by x.

(ii) Construct a right-angled triangle in which the sides forming the numerator and denominator of the ratio are of lengths x and 1 respectively and by means of the theorem of Pythagoras find the length of the third side.

(iii) From the figure read off the values of any of the other five ratios which are required and substitute in the expression considered.

The following examples will suffice to illustrate the method.

(*a*) Prove that

$$\sin A \;(\cot A + 2)(2 \cot A + 1) = 2 \csc A + 5 \cos A.$$

(i) Write $\cot A = x = \text{adj./opp.},$

(ii) and $\text{hyp.} = \sqrt{1 + x^2}.$

(iii) The equation is therefore equivalent to

$$\frac{1}{\sqrt{1+x^2}}(x+2)(2x+1) = 2\sqrt{1+x^2} + \frac{5x}{\sqrt{1+x^2}}$$

or

$$\frac{(x+2)(2x+1)}{\sqrt{1+x^2}} = \frac{2(1+x^2)+5x}{\sqrt{1+x^2}},$$

which is at once seen to be an identity.

(*b*) Simplify

$$\csc^2 A \tan A - 2 \sin A \cos A - \sin^2 A \tan A.$$

(i) Write $\sin A = x = \text{opp./hyp.},$

because though $\tan A$ occurs as often as $\sin A$, the reciprocal of the former, $\cot A$, is absent while the reciprocal of the latter, $\csc A$, is present.

(ii) Hence $\text{adj.} = \sqrt{1 - x^2}.$

(iii) Making these substitutions, we obtain

$$\frac{1}{x^2}\frac{x}{\sqrt{1-x^2}} - 2x\sqrt{1-x^2} - x^2\frac{x}{\sqrt{1-x^2}} = \frac{1 - 2x^2(1-x^2) - x^4}{x\sqrt{1-x^2}}$$

$$= \frac{1 - 2x^2 + x^4}{x\sqrt{1-x^2}} = \frac{(1-x^2)^{\frac{3}{2}}}{x} = \frac{\cos^3 A}{\sin A}.$$

Some of the first objections which naturally arise are as follows: (1) This method fails to give practice in using the

relations between the six ratios, (2) it is mechanical, (3) it is often clumsy.

In reply to which the following may be said: (1) It renders unnecessary the ordinary relations between the ratios only because it carries them back to their source in the definitions and the theorem of Pythagoras and to do this is a gain and no loss. (2) Economy of thought means replacing hard processes by simpler, more mechanical ones. The method is not mechanical in the sense that it hides or obscures the processes involved. (3) If we raise the treatment of identities from the uninstructive level of puzzle solving to the rank of sure and methodical solution a feeling of power is gained which may well balance an occasional clumsiness.

If the method is applied according to the preceding rules to the corresponding problems involving two or more angles it is undoubtedly clumsy; but a firm foundation has been laid for the treatment of those questions also. It has become clear that the identity of two expressions can not be immediately perceived until they are *both expressed in terms of the same units.*

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So long as we love, we serve; so long as we are loved by others I would almost say we are indispensable: and no man is useless while he has a friend.—*Robert Louis Stevenson.*